Tranformada de INTEGRales

CONVOLUCION de f por g : $f * g$

Para $f, g \in \mathcal{C}[0,\infty]$

$$f * g = \int_0^t f(\tau) g(t-\tau)\, d\tau$$

p.e. $e^t * \operatorname{sen} t = \int_0^t e^\alpha \operatorname{sen}(t-\alpha)\, d\alpha = \frac{1}{2}(-\operatorname{sen} t - \cos t + e^t)$

Ejercicio $f * g = g * f = \int_0^t g(\alpha) f(\underbrace{t-\alpha}_{u})\, d\alpha = \int_t^0 g(t-u) f(u)(-du)$

TEOREMA DE CONVOLUCION : $L(f) = F(s)$ $L(g) = G(s)$

$$\mathcal{L}(f * g) = \mathcal{L}(f)\mathcal{L}(g) = F(s)G(s)$$

Ejemplo $L\left\{\int_0^t e^\alpha \operatorname{sen}(t-\alpha)\, d\alpha\right\} = L e^t \, L \operatorname{sen} t$

$$= \frac{1}{s-1} \cdot \frac{1}{s^2+1}$$

Tambien $f * g = L^{-1}(F(s)G(s))$

Ejemplo: Calcular $L^{-1}\left\{\frac{1}{(s^2+k^2)^2}\right\}$

como $L^{-1}\left(\frac{k}{s^2+k^2}\right) = \operatorname{sen} kt$

CONT (2)

Con $F(s) = G(s) = \dfrac{1}{s^2+k^2}$ se tiene

$$f(t) = g(t) = \frac{1}{k} \cdot \mathcal{L}^{-1}\left\{\frac{k}{s^2+k^2}\right\} = \frac{1}{k}\,\operatorname{sen} kt$$

así $$f * g = \mathcal{L}^{-1}\left\{\frac{1}{(s^2+k^2)^2}\right\} = \frac{1}{k^2}\int_0^t \operatorname{sen} k\alpha \,\operatorname{sen} k(t-\alpha)\, d\alpha$$

pero $\operatorname{sen} A \cos B = \dfrac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s^2+k^2)^2}\right\} = \frac{1}{2k^2}\int_0^t \left[\cos k(2\alpha - t) - \cos k\alpha\right] d\alpha$$

$$= \frac{\operatorname{sen} kt - kt\cos kt}{2k^3}$$

así $$\mathcal{L}(\operatorname{sen} kt - kt\cos kt) = \frac{2k^3}{(s^2+k^2)^2}$$

Transformada de UNA INTEGRAL

$$\mathcal{L}(f*g) = L(f)\,Lg = L\left(\int_0^t f(\alpha)\,g(t-\alpha)\,d\alpha\right)$$

Con $g=1$ $\quad L(g)=\frac{1}{s}$ $\quad$ RESULTA

$$L\left(\int_0^t f(\alpha)\,d\alpha\right) = \frac{L(f)}{s} = \frac{F(s)}{s}$$

$$\int_0^t f(\alpha)\,d\alpha = L^{-1}\left(\frac{F(s)}{s}\right)$$

Ejemplo

$$L^{-1}\left(\frac{1}{s(s^2+1)}\right) = L^{-1}\left(\frac{\frac{1}{s^2+1}}{s}\right) = \int_0^t \operatorname{sen}\alpha\,d\alpha = 1-\cos t$$

$$L^{-1}\left(\frac{1}{s^2(s^2+1)}\right) = L^{-1}\left(\frac{\frac{1}{s(s^2+1)}}{s}\right) = \int_0^t (1-\cos\alpha)\,d\alpha = t-\operatorname{sen} t$$

$$L^{-1}\left(\frac{1}{s^3(s+1)}\right) = \int_0^t (\alpha-\operatorname{sen}\alpha)\,d\alpha = \frac{1}{2}t^2 - 1 + \cos t$$

UNA ECUACION INTEGRAL

Resolver $f(t) = 3t^2 - e^{-t} - \int_0^t f(\alpha)\, e^{t-\alpha}\, d\alpha$ para f

$$L(f) = 3L t^2 - L e^{-t} - L \int_0^t f(\alpha) e^{t-\alpha}\, d\alpha$$

$g(t) = e^t$

$$F(s) = \frac{6}{s^3} - \frac{1}{s+1} - F(s)\cdot\frac{1}{s-1}$$

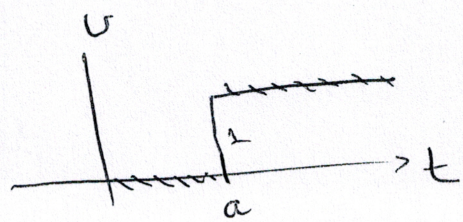
$$\therefore F(s) = \frac{6}{s^3} - \frac{6}{s^4} + \frac{1}{s} - \frac{2}{s+1}$$

$$y \quad f(t) = 3t^2 - t^3 + 1 - 2e^{-t}$$

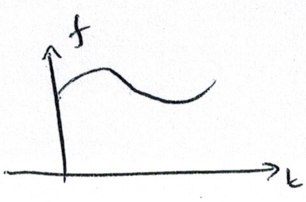
TEOREMA $F(s) = L(f(t))$, $a > 0$ entonces

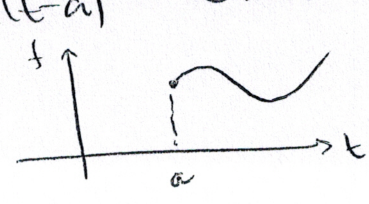
$$L\{f(t-a)\,U(t-a)\} = e^{-as} F(s)$$

con $U(t-a) = \begin{cases} 0, & 0 \le t < a \\ 1 & t \ge a \end{cases}$



$$y \quad f(t-a)\,U(t-a) = \begin{cases} 0, & 0 \le t < a \\ f(t-a) & t \ge a \end{cases}$$





Use T41
$$\begin{cases} F(s) = L(f) & n = 1, 2, 3, \ldots \\ L(t^n f(t)) = (-1)^n \dfrac{d^n F(s)}{d s^N} \end{cases}$$

para hallar

$L(te^{-10t})$ $L(t^3e^t)$ $L(t\cos 2t)$ $L(t\,\text{senh}\,3t)$

$L(t^2\,\text{senh}\,t)$ $L(t^2\cos t)$ $L(te^{2t}\sin bt)$ $L(te^{-3t}\cos 3t)$

Resolver

$$y' + y^{*} = t\,\text{sen}\,t \qquad y(0) = 0$$

$$y' - y = te^t\,\text{sen}\,t \qquad y(0) = 0$$

$$y'' + 9y = \cos 3t \qquad y(0) = 2,\ y'(0) = 5$$

$$y'' + y = \text{sen}\,t \qquad y(0) = 1,\ y'(0) = -1$$

En $ty'' - y' = 2t^2$, $y(0) = 0$

$$2y'' + ty' - 2y = 10 \qquad y(0) = y'(0) = 0$$

Use el teorema T41

para reducir la e.d.ó. dada en una CONT(6)

ecuacion lineal de primer orden para

$\mathcal{L}(y(t)) = Y(s)$ y despues encuentre

$y(t) = L^{-1}(Y(s))$

Use el teorema de convolución para hallar

$L(1 * t^3)$ $L(t^2 * te^t)$ $L(e^{-t} * e^t \cos t)$

$L(e^{2t} * \operatorname{sen} t)$ $L(\int_0^t e^{\alpha} d\alpha)$ $L(\int_0^t \cos\alpha \, d\alpha)$

$L(\int_0^t e^{-\tilde{t}} \cos \tilde{t} \, d\tilde{t})$ $L(\int_0^t \alpha \operatorname{sen}\alpha \, d\alpha)$ $L(\int_0^t \alpha e^{t-\alpha} d\alpha$

$L(\int_0^t \operatorname{sen}\alpha \cos(t-\alpha) d\alpha)$ $L(t \int_0^t \operatorname{sen}\alpha \, d\alpha)$

$L^{-1}\left(\frac{1}{s(s-1)}\right)$ $L^{-1}\left(\frac{1}{s^2(s-1)}\right)$ $L^{-1}\left(\frac{1}{s^3(s-1)}\right)$ $L^{-1}\left(\frac{1}{s(s-a)^2}\right)$